# Practice Exam for the Midterm 1.

**Problem 1**

Let

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 5 & 3 \end{pmatrix}.$$

Determine the nullity and rank of the linear transformation associated to $A$.

**Problem 2**

Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by $T(\boldsymbol{i}) = (1,1)$, $T(\boldsymbol{j}) = (1,0)$, $T(\boldsymbol{k}) = (0,1)$. Take basis $\mathcal{A} = \{(1,0),(0,1)\}$ and $\mathcal{B} = \{(1,1),(1,-1)\}$.

1. Determine the matrix representation of T relative to the standard basis $\{\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}$ for $\mathbb{R}^3$ and $\mathcal{A}$ for $\mathbb{R}^2$.
2. Determine the matrix representation of T relative to the standard basis for $\mathbb{R}^3$ and $\mathcal{B}$ for $\mathbb{R}^2$.

**Problem 3**

Let

$$A = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}.$$

1. Determine the inverse matrix $A^{-1}$.
2. Solve the following system of equations:

$$3x + 2y = 2, 4x + 3y = 5$$

**Problem 4**

Let

$$A = \begin{pmatrix} 5 & 6 \\ -4 & -5 \end{pmatrix}.$$

1. Determine all of the eigenvalues of $A$.
2. Find an invertible matrix $C$ such that $C^{-1}AC$ is a diagonal matrix.

**Problem 5**

Let $f$ be defined by

$$\begin{cases} \frac{x^2y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0). \end{cases}$$

1. Prove that $f$ is continuous over $\mathbb{R}^2$.
2. Determine the directional derivative $f'(p;\boldsymbol{v})$ at the point $p = (1,1)$ with respect to the vector $\boldsymbol{v} = (2,1)$.

**Problem 6**

Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ and $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = (\sqrt{xy}, x^2+y^2)$ and $g(u,v) = e^{uv}$.

1. Compute the Jacobian matrix of $f$
2. Let $h\colon \mathbb{R}^2 \to \mathbb{R}$ be the composition $g \circ f$. Express $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$ by using $x$ and $y$.